

Aber schon die Wurzel der beiden Worte, das indoeuropäische *dek-, bedeutete aller Wahrscheinlichkeit nach beides, sowohl empfangen wie grüßen. Die verwandten und aus derselben Wurzel stammenden lateinischen Worte *decet*, *decus* und *dignus* zeigen des weiteren, daß das Empfangene und Übernommene zugleich auch als das Wertvolle erfahren wurde, als das, was »paßt«, was »sich schickt«, daher auch »die Zierde« und »der Schmuck« ist, wie übrigens das Wort δόξα in der späteren griechischen Sprache immer mehr die Bedeutung von »Ansehen« und »Würde« angenommen hat.

Für die Slawischsprechenden ist es von besonderer Wichtigkeit, daß aus derselben Wurzel *dek- auch das altslawische Verb *desiti* stammt, das ursprünglich »finden« und »vorfinden« bedeutet, abgeleitet aber – und zwar offensichtlich vor allem im südslawischen Sprachgebiet – auch »geschehen«, »sich ereignen« sowie »begegnen«. In der nominalen Ableitung *des* für das zugeteilte Glück (im Sinne vom lateinischen *fatum*), und *udes* für das Schicksal, Verhängnis, Unglück, wird also vor allem die Begegnung zum Ausdruck gebracht, dann das zugeneigte Empfangen und Aufnehmen des Begegneten bzw. Geschickten, und zuletzt dessen Bewahren und Hüten. Im Einklang damit bedeutet das weiter abgeleitete Verb *udesiti* vor allem das In-Ordnung-bringen, und daher auch das Schmücken, während das Adjektiv *udesan* die Bedeutungen »angemessen«, »anmutig« und »gelegen« hat.

Belassen wir es bei diesen Hinweisen und enthalten uns eines jeden leichtfertig gezogenen Schlusses. Die Frage, was ist *Doxa*? bleibt einstweilen ohne Antwort. Die kaum zu überschätzende Bedeutsamkeit dieser Frage liegt darin, daß nur die Antwort auf ihr uns vermutlich zur angemessenen Beurteilung der Grundannahme verhelfen könnte, die auch lange schon vor Husserl sowohl die Philosophie wie alle wesentlichen Bahnen unserer Geschichte leitet, der Annahme nämlich, es gebe im ganzen neuzeitlichen Leben keine mächtiger und unaufhaltsamer vordringende Idee als die der Wissenschaft.

Vielleicht haben wir aber einen Schritt schon damit getan, daß wir durch diese Überlegungen ermutigt sind anzunehmen, bei dieser Annahme handle es sich am Ende doch nur um eine Annahme und nichts mehr.

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CAN WE PERCEIVE MATHEMATICAL ENTITIES?

There is the old question about the mode of existence of mathematical entities. Penelope Maddy (1980) in her article »Perception and Mathematical Intuition« attempts to defend, as she says, one kind of Platonism – a realism concerning the independent existence of mathematical entities. In fact, Maddy prefers the term »realism« rather than »Platonism«; she says that »set theoretic realism is a view whose main tenets are that sets exist independently of human thought, and that set theory is the science of these entities« (Maddy 1980 p. 163). But the term »realism« is applied with only the slight difference which is not so important for general ontological claims dealt in her and in this article so I will stick more often to the more traditional term of »Platonism«. The difference is only that Maddy (1980, p. 163, footnote 1) takes sets to be individuals or particulars and not universals. So, Maddy wants to support some of Gödel's remarks about the independent existence of mathematical entities. (But, as we shall see, it is her own interpretation of his statements, and Gödel has to be excused from *her* interpretation.) Gödel claims that the assumption of independent existence of mathematical entities is quite as legitimate as the assumption of the physical bodies and there is quite as much reason to believe in their existence. Maddy (1980) and Charles Chihara (1982) call it the equi-supportive claim. The other claim of Gödel is that mathematical axioms force themselves upon us as being true. Accepting axioms as true, we accept also entities about which these axioms are. *A fortiori*, we accept also the theorems which follow from the axioms accepted. We grasp these through the mathematical intuition. These entities about which axioms and theorems are about, according to Gödel and other (»pure«) platonists, are not spatio-temporal entities but abstract entities which have

independent existence. They would exist even if we, or even if nobody (except God, of course), never knew about them.

But, it is the fact that we know about them; so the problem is how these entities, if they are not spatio-temporal, affect us that we know what we know about them? How can something, what is not spatial and temporal, have an influence on spatio-temporal beings in spatio-temporal universe? How can mathematical entities influence our mathematical intuition? What is the nature of *connection* between independent mathematical entities and mathematical intuition?

These difficult questions laid down, Maddy wants to show that we can have knowledge about, for example, sets, in the way that is compatible with the causal theory of perception and knowledge. The causal theories of perception and knowledge are devised to deal with spatio-temporal entities in our physical spatio-temporal world. First, we have to show how we can accept a belief in the existence of physical objects. We form (perceptual) beliefs from perception. Maddy (1980, p. 171) accepts Pitcher's (1971) causal theory of (visual) perception and cites his conditions:

P perceives an X at I if and only if:

1. there is an X at I;
2. P acquires perceptual beliefs about X, in particular, that there is an X at I;
3. the X at I is involved in the generation of this perceptual belief state in an appropriate causal way (in the kind of way, for example, my hand is responsible for my perceptual belief that there is a hand before me when I look at my hand in good light)

In some detail, this process goes like this: the object reflects light, the light is composed of photons which travel to the eyes; the eyes have lenses which refract the light and transform the light into the electrical impulse at the visual nerve, and this impulse is sent by the neural nets to visual areas of the brain and so we have a state or process in the brain. Then, some neurophysiological mechanism develops which forms our perceptual beliefs from the acquisition of perception. Maddy (1980, pp. 178–179) at this point goes further and argues that we causally perceive *the sets of physical bodies*, and her example is the set of three eggs which has spatio-temporal characteristics:

What I want to suggest now is simply that we do acquire perceptual beliefs about sets of physical objects, and that our ability to do this develops in much the same way as that in which our ability to perceive physical objects develops, as described in the previous section. Consider the following case: P needs two eggs for a certain recipe, reaches into the refrigerator for the egg carton, opens it, and sees three eggs there. This belief (that there are three eggs before P) is perceptual because it is an integral part of the body of beliefs making up (or perhaps partly constituting) P's perceptual state. Other perceptual beliefs acquired on this occasion probably include details about size and color of the eggs, the fact that two eggs can be selected from among the three in various ways, the locations of the particular eggs in the nearly empty carton, and so on. The numerical beliefs are clearly part of this complex of perceptual beliefs because they can influence the others as well as being influenced by them. (For example, the welcome fact that there are enough eggs for the recipe can make the eggs themselves look larger.) So, the various numerical beliefs acquired on this occasion are perceptual and I further claim that they are beliefs about set, that is, I claim P acquires the perceptual beliefs that there is a set of eggs before P, that it is three-membered, and that it has various two-membered subsets.

There are, however, also sets of, for example, odd or real numbers etc., which are not accessible to perception, to such causal relations, because, as Maddy (1980, p. 179) herself admits a little bit further, they lack physical properties and spatio-temporal location. So, the problem is not yet solved. Nobody perceives, nobody ever reported honestly and convincingly, that he or she had perceptual acquisition of the set or even of a small subset of, for example, real numbers. This set does not come into existence or ceases to exist at some point, nor does it move around as we can say for (the set of) three eggs. In fact, at this point, Maddy's ontology is even more complicated than usual ontologies because she increases the number of kinds of existing entities. Instead of physical objects only (nominalism), or physical objects and abstract objects, now we have ontology composed of:

- physical objects
- sets of physical objects
- abstract mathematical entities (pure sets)

We can also say that Maddy has not attempted to reduce abstract mathematical entities to physical objects, because it would be a case of nominalism, not Platonism. We can ask now how can we distinguish between three eggs and the set of three eggs? Charles Chihara (1982, pp. 223–224) offered a fatal criticism of this point:

Imagine that I am sitting at my desk. Its surface has been cleared of everything except an apple. Now according to Maddy, we can literally perceive on the desk, in addition to the apple, the set of apples on my desk (which happen to be a unit set). It is claimed that this set has a location in space, the exact spot where the apple is. Supposedly, it also came into existence at a particular time (when apple did), and will go out of existence at particular time (when the apple does). Obviously, Maddy thinks this set can be moved about in space. Now if we can perceive this set with the sense of sight, then what does it look like? Evidently, it looks like the apple itself. After all, I don't see anything at that exact region in space that looks different from the apple. One wonders how this object is to be distinguished (perceptually) from the apple, since it has exactly the same shape and color. Perhaps it feels different. Let's touch it. But I can't feel anything there other than the apple. How about its smell or taste? Again, it would seem that the set must be indistinguishable in smell and taste to the apple. So it looks, feels, smells, and tastes exactly like the apple and is located in exactly the same spot and at exactly same time – yet it is a distinct entity!

Also, if we perceive sets of physical objects, we can perceive their units, disjunctions, etc., because they are also sets (of physical objects). Look at the case in which we have three eggs in a carton which is placed on the table (I shall ignore the other possible surrounding). We perceive three eggs, one carton and one table; one set of three eggs, one set of one table (unit set), and one set of a carton (also a unit set). Next, we perceive the union of a set of eggs and set of table; the union of the sets of eggs and the carton; the union of the sets of eggs, table and carton. We should perceive all their subsets also, etc., and if the sets listed above are proper sets as Maddy wants them to be, we should somehow perceive or be able to be acquainted perceptually with an empty set, because empty set is a subset of every set. But what would that be? When we are in above situation observing eggs in a carton laid on the table, is the »perceived« empty set a set of dolphins or elephants or planets we do not perceive at that moment? So instead of five bodies, we perceive a number of entities as Maddy tells us! But the whole situation can well be described by involving only five physical objects perceived. We haven't improved our theory if perception of sets of physical objects is involved. I think that these reasons given above are strong claims for rejecting Maddy's theory.

Throughout Maddy's article we also cannot find any part describing how it could be possible that *abstract entities causally affect us*. But if she wants to defend *Platonism* with the aid of causal theory of perception/knowledge, she has to provide an answer to this crucial question! I

think that nobody perceives, sees, touches the set of real numbers, or sets of mathematical functions etc., like we can see or touch eggs, tables and apples. Even if we accept (but I don't) that we perceive *sets* of physical entities, the problem of causal influence of abstract mathematical objects remains.

We can ask with what sense we should be able to perceive abstract mathematical entities, if we want to have a theory concerning abstract mathematical objects compatible with the causal theory of perception and knowledge, as Maddy would like to have it. We cannot say that the set of real numbers reflects the light which our eyes refract and... (see other details above). Nobody has ever reported that he or she literally saw such a set or any other abstract mathematical entity. Nor do any other sense *perceive* abstract objects. Further view I do not attribute to anyone, but somebody may perhaps suppose that mathematical abstract entities causally interact directly with the brain. But it seems highly implausible; and it is difficult to see what reasons and/or empirical evidence could be given to support such a claim.

Even if abstract mathematical entities exist just as we formulate them, but we cannot have a causal interaction with them, the causal theory of perception and knowledge cannot be the ground for supporting realist claims. Our mathematical theories are, then, at best, guesses or we know about them in some other way which is not a causal way and causal theories cannot justify their independent existence even if that objects exist. We should have a different theory and kind of knowledge to justify the independent existence of abstract mathematical entities – sets – if this is the case.

As far as Maddy's theory is concerned, we can easily see that she hasn't offered a successful justification of *Platonism* or, as she prefers, *realism*. Moreover, she has in fact introduced a new category of sets, i.e. the sets of physical objects, which are in fact unnecessary because we cannot distinguish between physical objects and sets of physical objects; and »set of physical objects« complicate the picture of what we perceive and the ontological picture of what exists and in what way. We can explain and dispense with physical objects without involving new ontological category of »sets of physical objects«. Moreover, *sets* of physical objects are also different from abstract mathematical sets.

Perhaps Maddy wants to suggest that the formation of our ideas and concepts about mathematical entities is causally dependent on the formation of ideas and concepts of physical objects. I would not ascribe

what follows that it is her opinion, but I would like to examine this possibility. Last sentence in her article encourages me to do that: »In particular, I have argued that if sets exist, we can know about them and refer to them in ways that should be acceptable to the causal theorist« (Maddy 1980, p. 196).

We perceive physical objects and form perceptual beliefs in the described causal way. Maddy has proposed that we form, in a basically similar way, perceptual beliefs about *sets* of physical objects. Maybe she would like to suggest, though it is not explicitly stated in her article, that we causally form the ideas and concepts of abstract mathematical entities, like sets for example, from perceptual beliefs about non-abstract (»impure«) sets, i.e., sets of physical objects (»set« conceived here meaning something more like an aggregation of objects rather than some independent (or) ontological category). She would probably also claim that this causal pattern is the justification of the causal theory of knowledge of mathematical entities, which therefore exist independently of our minds in their own reality. In other words, we do some further abstractions, which have some causal input from perceptual beliefs about »sets« (aggregations) of physical objects. Then we do more and more abstractions to formulate beliefs about something more abstract what we call pure mathematical entities. But then, this would be just a description of how we psychologically reason about them. Causal link in this case does not start from mathematical sets but from physical objects and aggregations (»sets«) of them. Causal link should start from mathematical (abstract) sets if causal theory would have to be support for our knowing about them and for the claim about their independent existence. The things are turned around: mathematical sets are at the end of a process and they should be at the beginning of the causal process. It seems rather that the creatures of that further abstraction processes are mind-dependent and so that this way of reasoning would go further in the direction of conceptualism rather than realism.

In conclusion, I would like to say (along with Chihara) that we should reject Maddy's theory and that it is even more complicated than traditional Platonism. In fact, Maddy hasn't offer a defence of what she calls realism. It seems that it is not successful from both ontological and epistemological points. I have not argued that we should reject any form of Platonism or realism about mathematical entities, but I have argued that Maddy's interpretation of Gödel's Platonistic claims (which are in fact far from original Gödel's claims) is not a support of *Platonism* or so based Gödelian *realism*.

Literature

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