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The First Decade of the Logic of Change LCG

IVAN RESTOVIĆ
Institute of Philosophy, Ulica Grada Vukovara 54, 10000 Zagreb, Croatia
ivan@ifzg.hr

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ABSTRACT: LCG is a logic of change proposed in 2007 by Świętorzecka. Its initial goal was to analyze in a logical framework the Aristotelian theory of substantial change – a kind of change that is more fundamental than others. Motivated by Aristotle's account, Świętorzecka proposes to enrich the language of classical logic by two operators of change, reflecting the Aristotelian dichotomy of generation and corruption. However, along with the Aristotelian interpretation, as an independent formal system, LCG offers a plethora of possible interpretations and insights. In the present article I briefly summarize the existing work in LCG and reflect on its further development.

KEY WORDS: Aristotelian theory of substantial change, classical conceptions of changeability, epistemic changes, growing language, logic of change LCG, modality, temporal logic.

1. Introduction

The logic LCG, a variant of a logic of change, was proposed in (2007) by K. Świętorzecka. Its initial goal was to analyze in a logical framework the Aristotelian theory of substantial change. Aristotle proposed this kind of change to solve the old problem of the possibility of change itself. One of the persisting older views was that of Parmenides, who famously claimed that there was no change whatsoever. This counterintuitive view was the result of the infamous dilemma ascribed to Parmenides and described by Aristotle in (*Phys* 191a30-31). There are only two ways in which something can come-to-be. Either from what already is, or from what is not. This is the dilemma. What makes it infamous is that neither option is possible. Therefore, nothing can come-to-be. Aristotle tackles this problem by proposing a new kind of change, substantial change, thus dissolving the dilemma. It is beyond the scope of this article to give a full characterization of Aristotle's theory of change. Aristotle's philosophy plays only an inspirational role in the formalism of LCG, given most elaborately in (Świętorzecka 2008b). The logic of change LCG does not purport to fully represent all of Aristotle's views on change. Of course, there are some facets of LCG directly inspired by Aristotle's insights. Nevertheless, LCG, considered as an independent formal system, offers a plethora of possible interpretations and applications to philosophical problems.

The subjects of change in the Aristotelian framework are *individual substances*. These substances undergo various kinds of change, most notably *substantial change*. We speak about substantial change when one substance becomes another. The components of substantial change are *disappearing* (*destruction*) and *coming-into-being* (*generation*) (Świętorzecka 2008b). "[I]n substances, the coming-to-be of one thing is always a passing-away of another, and the passing-away of one thing is always another's coming-to-be" (cf. *GC* 319a20-22; *Met* 994b5-6; *Phys* 208a9-10; cited in Świętorzecka 2008b: 15). Looking to analyze this account in the logical setting, a couple of philosophical and logical remarks are in order. They are given most notably in (Świętorzecka 2008b) which further develops the account proposed in (Świętorzecka 2008a).

Firstly, one has to consider the very subjects of change. Aristotle speaks about individual substances undergoing change. The language of LCG is a propositional language enriched by one primitive operator of change. So, in LCG the subjects of change are situations. Aristotle talks about *things* coming to be or disappearing, while we talk about *situations* in which things come to be or disappear. These situations are expressed by *formulas* or *sentences* of logic LCG. Some further justifications of this approach are given in (Świętorzecka 2008b).

Secondly, there is the issue of the very ability of the system to simulate situations appearing and disappearing. The usual propositional languages are not designed to express such a dynamicity. In classical logic, the true propositions are usually just given and nothing changes. In standard temporal approaches there is change, but it is not substantial in a sense that sentences may change only their truth values. Still, the set of atomic sentences remains constant. In LCG we have a constantly growing set of atomic sentences. More on that in the second chapter.

There are also some more typically philosophical remarks which this logic of change wants to consider. (Świętorzecka 2008b) gives special attention to the notions of time and continuity vis-à-vis change. Following Aristotle, changeability-in-general is assumed to be ontologically independent of time. Generation and destruction "[…] are not even measurable in time since it is precisely the existence of time that is dependent on the existence of substance […]" (Świętorzecka 2008b: 13). This will be reflected in the formalism, where the operator of change C is chosen as primitive. As we will see later, this becomes important when comparing LCG to other formal systems.

Regarding the continuity of change, substantial changes are said to be *dichotomous.* This is reflected in the language of LCG. We are able to distinguish between different substances coming to be and disappearing at different stages of a growing universe[1]. All the changes occurring in LCG are discrete. Correspondingly, if we decide to adopt a temporal interpretation of change in LCG, we get a discrete branching temporal structure with the first element. Further remarks on this interpretation are given in (Czermak and Świętorzecka 2011). With the above philosophical considerations and commitments, we can turn to the formal description of LCG.

2. LCG Calculus

The following characterization is taken mostly form (Świętorzecka and Czermak 2012), where LCG is considered separately from Aristotelian philosophy. To get LCG we add to the language of propositional logic the primitive operator C, which reads "it changes that…". LCG has another important non-standard component, the notion of a *level of a formula*. Atomic propositions in LCG are indexed and form a set $\{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\}$. From a philosophical standpoint, we can call the elements *elementary situations* (Świętorzecka 2008b), keeping in mind the considerations about the subjects undergoing change given in the Introduction. Let us consider an example. The formula $\alpha_3 \rightarrow (\alpha_1 \vee \alpha_7)$ is said to be of the *minimal level* of 7. This number is the greatest index of an atomic subformula in the given formula. The level is *minimal* because the above formula can first appear only at the stage 7 of a development of a universe. Before that stage, it lacks truth value. But after that, it is always either true or false. Therefore, $\alpha_3 \rightarrow (\alpha_1 \vee \alpha_7)$ is of the minimal level 7, but also of the levels 8, 10, and so on.

To get the full axiomatic system of LCG, we take as axioms all classical propositional tautologies along with the four representative axiom schemata. Concerning the rules of inference, we have the usual modus ponens, the $\neg C$-rule and the replacement rule. The $\neg C$-rule simply states that theorems of the system do not change, formally: $A \vdash \neg CA$, where A is a theorem. The replacement rule says that if we have a formula A with a subformula B (nota-

[1] We can also consider an epistemic interpretation. More information on interpretations of LCG, as well as on the notion of a *universe,* is provided in chapters 4 and 3, respectively.

tion: $A[B]$) and it follows that B is equivalent to B' (formally: $B \leftrightarrow B'$), then we can replace the subformula B in A with B' (we can infer $A[B']$). Axiomatically, LCG is characterized by four axiom schemata:

Ax1) $CA \rightarrow C\neg A$

Ax2) $C(A \wedge B) \rightarrow CA \vee CB$

Ax3) $(\neg A \wedge B \wedge CA \wedge \neg CB) \rightarrow C(A \wedge B)$

Ax4) $(\neg A \wedge \neg B \wedge CA \wedge CB) \rightarrow C(A \wedge B)$.

Let us briefly consider the intuitive interpretation of the above schemata. For the present purpose, assume only that changes expressed by C are changes *to the truth value*. So, Ax1 says that if a formula changes its truth value, so will its negation. This, in my opinion, perfectly reflects our "naïve" understanding of change. The second axiom schema is also quite uncontroversial, describing the way in which we can distribute the operator of change. Finally, Ax3 and Ax4 may at first sight appear to lack the intuitive clarity of Ax1 and Ax2. Let us look at them together. They share the same consequent, namely $C(A \wedge B)$. This means that they both give us conditions for *changing the truth value* of $A \wedge B$. Now, both in the Ax3 and in Ax4 the conjunction $A \wedge B$ is not true in the antecedent. We could say that Ax3 and Ax4 provide us with the conditions for a change of a conjunction, given that the conjunction does not hold. Take Ax4. It says that if we do not have nor A nor B, to change their conjunction we have to change both elements.

I have previously said that the above four axiom schemata are characteristic of LCG. But C is not the only operator characteristic of LCG. While C lets us speak about changes to the truth value, the operator G allows us to speak about changes in *complexity* in a sense of *increasing the level* of a formula. With the help of the latter operator, we can acquire a growing language. As a matter of fact, G is a shorthand for two operators, $G^+$and G^-, which are introduced definitionally on the object-language level to reflect the phenomenon of a growing language. Let formula A be of minimal level $n-1$, then:

Def. G^+) $G^+A \leftrightarrow (A \wedge \alpha_n)$

Def. G^-) $G^-A \leftrightarrow (A \wedge \neg\alpha_n)$.

Let $\pm\alpha_n$ denote either α_n or $\neg\alpha_n$. Now we can understand G-changes as *growth* (which – incidentally – starts with a "g") in complexity or level of a formula. To say that a proposition underwent a G-change is to say that it was suplemented by a new proposition $\pm\alpha_n$. The proposition $\pm\alpha_n$ introduces either a positive or a negative "atomic situation". Remarks on the expanding language are given in (Świętorzecka 2012).

3. The semantics of LCG

The semantics of LCG has an important non-standard component, linked to the notion of a level of a formula. Let B_n denote the set of all possible conjunctions $\pm\alpha_1 \wedge\ldots\wedge \pm \alpha_n$. We understand B_n as the *universe* of the level n. The elements of a given B_n are understood as *possible worlds* (Świętorzecka 2008b). From all the possible conjunctions in a given universe, only one can be true. In other words, only one of the possible worlds in a given universe is actual. The notion of truth is defined with the help of the function φ, called a *history* of the development of a universe. When speaking about φ, we interpret the number n as the *stage* of a universe. For each stage n, $\varphi(n)$ chooses exactly one conjunction of the minimal level n. That conjunction is considered to be a *fact* in B_n, as opposed to other conjunctions, which are merely *fictions* (Świętorzecka 2008b). Moreover, as n rises, so does the number of possible worlds in a given universe. We start with B_1, where there are only two possible worlds: α_1 and $\neg\alpha_1$. At each new stage, the number of possible worlds is doubled. For every n there are 2^n conjunctions or possible worlds. This signifies a considerable dynamicity.

Consider therefore again the formula $\alpha_3 \rightarrow (\alpha_1 \vee \alpha_7)$. Having α_7 as a subformula, it can first appear at the seventh stage of the development of the universe. We know that the universe at stage 7 (or B_7) is a set of all possible conjunctions from $\pm\alpha_1$ to $\pm\alpha_7$. How many conjunctions are there in B_7? As it turns out: 2^7 = 128. At its seventh stage, the universe has 128 possible worlds, all of them of the length (or *complexity*) of 7. And in B_7 there is exactly one actual world $\varphi(7)$ and 127 *fictitious* worlds. Let us assume that $\alpha_3 \rightarrow (\alpha_1 \vee \alpha_7)$ is true in the actual world. This means that it follows from a seven-membered conjunction that "actually" holds (recall that this conjunction consists of atomic formulas and their negations). Fair enough, but it seems that $\alpha_3 \rightarrow (\alpha_1 \vee \alpha_7)$ can also follow from one of the 127 *fictitious* conjunctions. Surely there is some *alternative history* ψ in which $\alpha_3 \rightarrow (\alpha_1 \vee \alpha_7)$ follows from the axioms of LCG and a given conjunction of seven elementary formulas or negations thereof. Out of 127 worlds in B_7 which are not actual, how many of them make true the formula $\alpha_3 \rightarrow (\alpha_1 \vee \alpha_7)$? I will leave it to the reader to ponder about the answer – the point of this example is that the dynamic language of LCG opens new perspectives and questions which cannot be proposed in the setting of classical propositional logic.

Now we have the formal prerequisites to formally define the concepts of truth and validity in LCG. Regarding the truth conditions, for any atomic formula α_k $(1 \leq k \leq n)$ and complex formula A of level n:

i) $\varphi \models^n \alpha_k$ iff α_k occurs in $\varphi(n)$ without the sign of negation

ii) $\varphi \models^n CA$ iff ($\varphi \models^n A$ and $\varphi \not\models^{n+1} A$) or ($\varphi \not\models^n A$ and $\varphi \models^{n+1} A$).

If A is of the minimal level $n-1$:

iii) $\varphi \models^n G^+A$ iff $\varphi \models^n A$ and $\varphi \models^n \alpha_n$

iv) $\varphi \models^n G^-A$ iff $\varphi \models^n A$ and $\varphi \not\models^n \alpha_n$.

Truth conditions for the usual logical connectives are the same as in classical propositional logic, mutatis mutandis. Moreover and importantly:

*) If $n < lv(A)$ then $\varphi \models^n A$ is not defined, i.e. A lacks truth value at n (where $lv(A)$ signifies the minimal level of the formula A).

$\varphi \models^n A$ stands for "A is true at stage n of some universe". Previously, we adopted an intuitive interpretation of C-changes as changes to the truth value. Now we can see that we were initially correct. Truth conditions for G-changes are semantical counterparts of their definitions given in the previous chapter.

In the above conditions we see two deviations from the semantics of classical propositional logic. First is the case of atomic propositions. The second case applies to (nearly) all other propositions. The first case is the underlying structure of LCG, differing from the classical picture. It is composed out of conjunctions of atomic formulas and negations thereof. To say that α_k is true is to say that is located at the k-th place of the conjunction (which is $\varphi(n)$). We can picture atomic formulas forming "chains" of complex situations. The second deviation appears to be of both logical and philosophical importance. From the logical point of view, there can be cases when a proposition does not have a truth value. Recall the formula $\alpha_3 \rightarrow (\alpha_1 \vee \alpha_7)$. One cannot decide on its truth value before level 7. Can this be considered a deviation from the famous principle of excluded middle? This proviso can also be of philosophical importance. When asserting $\alpha_3 \rightarrow (\alpha_1 \vee \alpha_7)$ in B_3, are we just talking gibberish, or expressing a hitherto undecidable proposition?

The last non-classical feature of LCG is the notion of φ-validity. It is a weaker version of validity proper, which is in turn defined by the weaker notion. For any formula A:

Def. φ-val.) A is φ-valid iff $\varphi \models^k A$ for all k: $n \leq k$, where A is of the minimal level n

Def. val.) A is valid iff A is φ-valid for all functions φ.

Every φ is a *choice function*. For every consecutive n, it *chooses* one $\varphi(n)$. Now, the value of φ can be considered "random". At each stage of the universe, the choice function φ chooses one possible word. And if it happens for a formula to be as lucky as to be true in every consecutive possible world, it is called *φ-valid*. This is a weaker notion of validity, reserved not only for logical truths. The definition of validity proper mentions "all functions φ". In (Świętorzecka 2008b) remarks are given on different kinds of histories. Additionally, we can characterize the "rhythms" of changing truth values by introducing certain formulas as axiom schemata.

The calculus LCG is sound and complete with respect to the here presented semantics. The full proofs are given in (Świętorzecka 2008b). Completeness is proved using two different techniques – namely – using conjunctive normal forms and Henkin-style proof.

4. Applications and interpretations of LCG

Having a sound and complete formal system, its history ceases to matter. As a mathematical intuitionist would say – once constructed, a mathematical entity develops a life of its own, undergoing constant transformation. The hallmark operators of LCG offer a wide range of possible interpretations. In the last chapter we spoke about a growing universe. The successive values of $\varphi(n)$ can therefore have an ontological interpretation, inherited from the Aristotelian motivation. Alternatively, we can model growing sets of *beliefs* of some agent, or the stages of a development of a *proof* or *argumentation* (Świętorzecka and Czermak 2012). (Świętorzecka and Czermak 2015) gives a Leibnizian interpretation: the notion of *monads* and the relation of *compossibility* are expressed in the language of LCG. In (Restović 2017) LCG is used to formally analyze the philosophy of L. E. J. Brouwer.

Leaving Leibniz and Brouwer aside for another occasion, let us consider Parmenides and Heraclitus, as was done in (Świętorzecka 2009). As we have seen in the Introduction, Aristotle formulated his account as a response to his predecessors. So, it is only natural to explore what LCG as a modern tool can contribute to the discussion. In the setting of LCG, Parmenides' view can be expressed by the formula $\neg CA$. We can add this to LCG as an axiom schema to get a logic of Parmenides. In the case of Heraclitus, for whom change is all there is, we run into a problem. It turns out that there can be no "logic of Heraclitus", which would be intuitively characterized by the axiom schema CA. The more obvious reason for this is the $\neg C$-rule of LCG, which makes sure theorems do not change. If everything were to change, so would the theorems, but they cannot, since they are theorems. We have an immediate contradiction. So, change is not *all* there is, but could contingencies (constantly) change? If we add CA as an axiom schema, and reserve A only for contingent formulas[2], we still get a contradiction. A simple proof is given in (Świętorzecka 2009).

Speaking of contradictions, LCG is also applied to the two famous ancient paradoxes. In (Świętorzecka 2009) LCG is used and slightly semantically modified in order to express the liar's paradox. It is shown that in LCG the characteristic self-referential sentences of a liar are not strictly paradoxi-

[2] Contingent formulas are such formulas that are not theorems of the system, but neither are their negations theorems.

cal. They merely have oscillating truth values. In (Czermak and Świętorzecka 2011) the language of LCG is used to give a new perspective on the nature of time in the setting of Zeno's paradoxes. As noted above, if we decide to give LCG a temporal interpretation, we get discrete time. Świętorzecka and Czermak note that this is in opposition to the view which Zeno is committed to. And speaking of oppositions, LCG can also contribute to the long-lasting discussion about the square of oppositions. In (Świętorzecka and Czermak 2010) the changing truth values expressible in LCG are associated with geometrical objects like squares and cubes.

Having reflected on the tradition, it is only when considering LCG as an independent formal system that we start uncovering its full potential. (Godlewski, Świętorzecka and Mulawka 2014) joins the efforts of artificial intelligence in implementing more complex logical systems in a machine. There is given a computer method of finding valuations forcing validity of LCG formulas. Already in (Świętorzecka 2009) LCG is compared to a logic of A. Prior (1957). He developed a logic with a temporal operator, to be able to speak about the future. Some parallels can be drawn between LCG and Prior's system. In a way, LCG also speaks about the future, but only indirectly. Asserting CA is asserting that A will change *in the future*, but "the future" is not a primitive concept. In (Świętorzecka 2008b) an alternative calculus is proposed, replacing the operator C with the operator N, which reads: "the next is that …". All sentences with C's can be translated into sentences only with N's.[3] Naturally, different axioms are needed. But all in all, it follows that LCG with the operator N is equivalent to Prior's calculus, which is again equivalent to that of von Wright (1965), as shown by Clifford (1966).

The inter-definability of time and change in a formalism is an interesting topic in-and-of itself. For instance, in a frame of formal ontology, the decision about the priority or posteriority of time vis-à-vis change can be the very starting point. There is even a middle ground. We can take both C and N to be primitive in our system, thus extending the vocabulary of the logic of change.

5. Extensions and ongoing research

A modal extension of LCG is given in (Świętorzecka and Czermak 2015). The language of LCG is enriched by the two characteristic modal operators $\Box$ and $\Diamond$ to obtain the system LCS4. The notion of necessity is introduced, allowing us to speak about a formula being true in *all* the next stages. We had something similar before, but in the meta-theory. LCS4 introduces a coun-

[3] $CA \leftrightarrow (A \leftrightarrow \neg NA)$; $\varphi \models^n NA$ iff $\varphi \models^{n+1} A$.

terpart of φ-validity on the object-language level, endowing $\Box$ with the truth condition that, if $lv(A) \leq n$, then

v) $\varphi \models^n \Box A$ iff $\forall_{k \geq n} \varphi \models^k A$.

Given the axiomatic characterization of LCS4, in it we can derive the characteristic axiom of S4.3: $\Box(\Box A \rightarrow B) \vee \Box(\Box B \rightarrow A)$. The system LCS4 is also sound and complete.

An alternative extension of LCG was proposed by M. Łyczak in (2017). He introduces the operator *B*, to account for the phenomenon of changing beliefs. With *C* and *G* in stock, he proposes a logic that will capture in a unique way the dynamics of changing beliefs, thereby giving a new perspective distinct from the traditional approach of (Alchourrón, Gärdenfors, and Makinson 1985) or (Segerberg 1999). At the time of writing this article, "the logic of changing beliefs" is still under development, and so is LCG – being the formal basis. To sum up – in its first decade LCG has proven to be a fruitful starting point for developing new logical systems, as well as a powerful tool for logical analysis of both philosophical and logical problems.[4]

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